Evaluation of the Waiting-Time effect on Critical Gaps at Roundabouts by a Logit Model

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Several studies have suggested that the entry capacity of roundabouts depends on the critical gap. Accordingly, the assumption in calculating the capacity is that all drivers are homogeneous and consistent; i.e., their behavior does not change over time. This paper examines the accuracy of this assumption; in particular, it evaluates the effect of waiting times on drivers’ critical gaps.

The paper presents a new behavioral approach to estimate the impact on critical gaps of waiting time prior to entry into a roundabout. A disaggregate logit model is developed to study the effect of waiting time at an approach to a roundabout on the likelihood of accepting different gaps and, therefore, on the critical gap.

The estimated model showed that the waiting time has a significant effect on the critical gap, particularly on gaps in the range of 2 to 5 seconds. The importance of this model is that it shows quantitatively the reduction in the critical gap with the increase in waiting time. Therefore, roundabout capacity for this range of critical gaps is higher than that currently proposed by the Highway Capacity Manual (HCM 2000).
1. INTRODUCTION

In many European and Australian cities, roundabouts are becoming a practical, well-accepted solution to urban and suburban intersections. In the U.S. they have yet to become widespread: local and state planners have not familiarized themselves with the advantages of roundabouts, particularly in regulating traffic flows at low to moderate volumes. Other advantages are as follows: (a) added safety because of the need to reduce speeds on the main road; (b) enabling of a higher entry capacity from the secondary road to the main road compared to conventional unsignalized intersections; (c) simplicity and consistency of operations; (d) enhancement of aesthetics through use of the central island for beautification of the intersection area.

The entry capacity of any unsignalized intersection, including roundabouts, is a key issue for traffic engineers. It determines delay and, therefore, expected waiting time, as well as queue lengths. Delay is the primary measure in determining level of service and queue lengths, two factors that are important for a variety of engineering issues, such as length of turning lanes and possible blockage of access points. HCM 2000 (HCM, 2000) suggests that entry capacity is an exponential function that largely depends on the critical gap at the location where the capacity is calculated. Accordingly, to calculate the capacity, it is necessary to assume that all drivers are homogeneous; i.e., they all accept gaps larger than the critical gap and reject smaller gaps. The HCM also presupposes that drivers are consistent; i.e., that their behavior does not change over time. This paper examines the accuracy of the latter assumption; in particular, it evaluates the effect of waiting times on drivers’ critical gaps.

Assuming that drivers who approach a roundabout face a choice of accepting a given gap or rejecting it, this study utilizes a binary disaggregate choice model to study drivers’ behavior in accepting gaps at roundabouts. The effect of waiting time on the drivers’ decision is a behavioral issue, and therefore a behavioral model that provides a means of studying different variables that affect drivers’ behavior is used. Specifically, this study utilizes the logit model to study the effect of drivers’ waiting time on the approach to a roundabout on the likelihood of accepting different gaps and, therefore, on the critical gap.

2. LITERATURE REVIEW

Most roundabout entry-capacity models are based on the drivers’ gap-acceptance process and on the representative critical gap of the population. However, most gap-acceptance or rejection models do not consider the impact of the variability of gaps on capacity or the effect of waiting time on the acceptance process. In England, Ashworth (1969) showed that if a single gap-acceptance “step function” is replaced with a distribution of critical gaps, a considerable reduction would result in the capacity of the minor approach. Polus et al. (1996), who evaluated the learning rate of drivers on the minor road approach to an unsignalized intersection, discussed the possible reduction in critical gaps because of drivers’ becoming impatient as waiting time increase. They developed an s-shaped model to show gap deterioration over time; the critical gap was defined as the intersection of the accumulated distribution of accepted and rejected gaps.
Other previous prominent studies on entry capacity at roundabouts include Kimber (1980) in England, who proposed a regression model, based on several independent parameters to estimate the capacity; a Swiss study by Simon (1991); and a German study by Stuwe (1991). The Australian method, based on an exponential distribution of gap-acceptance functions calibrated in several studies, is presented in Austroads (1993). An Israeli model was developed by Polus and Shmueli (1997), the independent parameters being the outside diameter and the conflicting volumes. They found, similar to the German model but unlike the other models, which descend toward zero at high circulating flows, that the entry capacity did not approach zero in reality. Some minimum entry capacity on the approach leg is still available even at higher circulating flows, because waiting vehicles will generally accept smaller gaps after a long wait. Gap acceptance theory has also been used to better estimate the capacity of ramp weaves (Lertworawanic and Elefteriadou, 2002). Gap acceptance has also been studied in the safety literature. For example Herslund and Jorgensen (2002) studied the differences in gap acceptances in roundabouts if the gap is followed by a car or by a bicycle; Alexander et al. (2002) studies the various factors affecting gap acceptance and their influence on the risk of accident; Cooper and Zheng (2002) who studies the effects of in-car phone use on gap acceptance decisions, and Hammond and Horswill (2001) who studies the effects of drivers' characteristics on gap acceptance.

The logit model has been used in a few cases to study highway-design issues. Some of these studies used the logit model for better understanding of safety issues. Lee and Mannering (1999) employed a nested logit model to study the effect of roadside features on run-off-roadway accident severity; Chang and Mannering (1998) used accident data to estimate a nested logit model of vehicle occupancy and accident severity.

On the specific topic of gap acceptance, Mahmassani and Sheffi (1981) used a probit model to investigate gap acceptance and showed that the critical gap of drivers is decreasing on the average, as they are waiting for an acceptable gap. Daganzo (1981) used the probit model to determine simultaneously the mean critical gap, the mean critical lag (the first gap considered by a driver), and the variance of these and found that the mean critical gap was significantly smaller than the mean critical lag, as one might expect. Teply et al. (1997) in a two part article used a binary logit model to investigate driver gap-acceptance behavior at an unsignalized intersection. Their analysis considered the nature of opposing traffic, including time gap, space gap, speed and type of opposing vehicle, delay to vehicles turning across the traffic, including queue delay and front-line delay. They also considered driver characteristics (gender and age), acceleration capability of the turning vehicle, and the presence of vehicles behind the turning vehicle. The results showed that using the time gap alone might yield a reasonable practical approximation in an engineering analysis of entry behavior at unsignalized intersections, including roundabouts.

Pant and Balakrishnan (1994) developed a neural network for predicting the gap-acceptance behavior of drivers at rural, low-volume, two-way, stop-controlled intersections and compared it to results obtained from a binary-logit model. Taylor and Mahmassani (1998) developed probit models for both motorists’ and cyclists’ gap-acceptance behavior and found that both cyclists and motorists required a longer gap when the gap was closed by a large
vehicle (e.g. a bus), and both would accept a shorter gap when the gap was closed by a bicycle, relative to a gap closed by a private car.

3. METHODOLOGY

The present study employed the random-utility theory for the binary choice case to estimate the critical gap at roundabout intersections. For this purpose, a disaggregate model of gap acceptance was estimated. In order to estimate the critical gap based on this model, it was necessary first to define the critical gap on a disaggregate basis. Following Drew’s (1968) definition of the critical gap as the gap for which an equal percentage of waiting drivers will accept a smaller gap as will reject a larger one, it is possible to define the critical gap at a disaggregate level as the gap that has an equal probability of being accepted or rejected. This definition is a disaggregate definition, which means that the critical gap can vary from driver to driver and from gap to gap. Approaching the problem from a practical point of view and following Teply et al.’s (1997) conclusion that using the time gap alone might provide a reasonable practical approximation in engineering analyses at intersections, we considered only the effect of time gap, in addition to waiting time, which is the main focus of this study. It will be shown that waiting time has a significant effect on drivers behavior and, consequently, on the critical gap.

A driver waiting at the entrance to a roundabout faces a series of binary choices. For each gap, the driver has to decide whether to accept it and enter the roundabout or to reject it and wait for the next gap. This process continues until the driver accepts a gap and enters the roundabout.

For each case, the driver has a utility from accepting or rejecting a given gap. The utility from accepting a gap results from the avoidance of further delay at the roundabout, whereas the utility from rejecting a gap is the added safety resulting from not accepting a short, dangerous gap. The variables that affect these utilities and the decision whether to accept or reject a gap include the time (or space) gap, speed and type of conflicting vehicle in the roundabout, driver characteristics and vehicle characteristic, waiting time previous to the decision, and the overall design characteristics of the roundabout.

This paper focuses on the effect on the critical gap of the waiting time at the approach to the roundabout. The initial hypothesis is that the longer the driver waits for a gap, the more he or she might be willing to accept more risk and therefore, a shorter gap. Consequently, the drivers’ critical gap should decrease; concomitantly, the critical gap of the population would be smaller than at roundabouts where the average waiting time was relatively small and, therefore, had little, if any, impact on the gap acceptance behavior of drivers.

This relationship can be expressed as:

\[ t_{cr} = f(t_w) \]  

\( t_{cr} \) The critical gap in seconds
\( t_w \) The waiting time at the approach (including waiting time in the queue and waiting time at the stop line), in seconds
For this research, we chose to use the logit model because of its mathematical simplicity. The development of the logit model follows.

### 3.1 The Logit Model

The model is specified as a binary logit model in which the utility of each alternative response (accept or reject a gap) is specified as:

\[ U_i = V_i + \varepsilon \]  

where:
- \( U_i \) is the utility of alternative response \( i \) for a given traveler
- \( V_i \) is the systematic component of the utility
- \( \varepsilon \) is its random component

The systematic component of the utility can be written as:

\[ V_i = \beta' X \]  

where \( X_i \) is a vector of attributes for alternative \( i \), and \( \beta \) is a vector of coefficients.

In the logit model, \( \varepsilon \) are independently and identically Gumbel distributed (Ben Akiva, and Lerman, 1985). The probability that alternative \( i \) will be chosen is given by:

\[ p(i) = \frac{\exp(\mu V_i)}{\sum_{i \in L} \exp(\mu V_i)} \]  

where \( \mu \) is the scale parameter, and \( L \) is the set of available alternatives.

The roundabout-entry issue presents two alternatives: accept a gap or reject it. In the case of a linear utility function, the parameter \( \mu \) cannot be distinguished from the overall scale of the \( \beta \)'s, and therefore it is omitted from the utility function (Ben-Akiva and Lerman, 1985)

### 3.2 Data Collection and the Sample

Data were collected at seven busy urban and suburban roundabouts in Israel. The data collection was performed by videotaping the intersection from a hidden point above the intersection; the data was later reduced in the laboratory by analyzing each frame, at 0.1 seconds intervals, of the videotape. The data included, in addition to the circulating volumes, the waiting times on the approach leg from the drivers’ arrival till their entry into the circulating road, as well as the rejected and accepted gaps. The traffic control at all sites was a yield sign, as is common in roundabouts; because of the relatively large circulating volumes, however, the majority of drivers had first to stop and then perform the gap-acceptance process. Table 1 presents the outside diameter of the roundabout (from the island on one leg to the island on the opposite leg) and the range of circulating volumes.
Table 1. Traffic Characteristics and Geometry of the Seven Roundabout Sites Observed

<table>
<thead>
<tr>
<th>Site Number</th>
<th>Location</th>
<th>Outside Diameter (m.)</th>
<th>Circulating Volume Range (vph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Haifa</td>
<td>27</td>
<td>60-900</td>
</tr>
<tr>
<td>2</td>
<td>Ramat Hasharon</td>
<td>35</td>
<td>120-1020</td>
</tr>
<tr>
<td>3</td>
<td>Herzlia</td>
<td>29</td>
<td>120-1140</td>
</tr>
<tr>
<td>4</td>
<td>Kiriat Ata</td>
<td>27.5</td>
<td>60-660</td>
</tr>
<tr>
<td>5</td>
<td>Ramat Aviv</td>
<td>38</td>
<td>540-1500</td>
</tr>
<tr>
<td>6</td>
<td>Pardes Hana</td>
<td>36</td>
<td>300-1200</td>
</tr>
<tr>
<td>7</td>
<td>Tel Aviv</td>
<td>22</td>
<td>450-1400</td>
</tr>
</tbody>
</table>

When considering driver behavior at roundabouts, it is necessary to calculate those gaps that have an impact on the entry process. Long gaps, typically those that are longer than the time needed to travel along one quarter of the circle, are not meaningful, because vehicles are not required to consider them when making the decision whether to accept or reject a gap. Therefore, gaps longer than a certain threshold are not applicable to the gap-acceptance process as was discussed by Polus and Shmueli (1999). These thresholds were calculated for each roundabout based on its radius (taken from Table 1) and operating speed. The resulted threshold values ranges from 6.7 to 9.2 seconds and gaps larger than these values were eliminated for the model estimation.

From a data efficiency point of view, one should use all accepted and rejected gaps observed for each driver. However, one of the assumptions of the logit model is that the disturbances $\varepsilon$ are independently and identically Gumbel distributed. In order to be consistent with this assumption and to avoid dependency between observations for the same driver, one gap was randomly sampled for each driver. In this case, satisfying the independency assumption is more important than data efficiency.

The resulting sample consisted of 743 observations, of which 483 chose to accept a gap, and 260 to reject a gap. This sample was large enough to estimate the logit model with a high significance level while maintaining a reasonable data-collection budget. Table 2 provides the basic statistics of the data collected.

Table 2. Basic Statistics of the Data Collected

<table>
<thead>
<tr>
<th></th>
<th>Accepted Gaps, Sec.</th>
<th>Rejected Gaps, Sec.</th>
<th>All Gaps, Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>5.85</td>
<td>2.63</td>
<td>4.79</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.47</td>
<td>1.08</td>
<td>2.03</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Maximum</td>
<td>9.2</td>
<td>6.1</td>
<td>9.2</td>
</tr>
</tbody>
</table>
3.3 Model Estimation

The deterministic components of the utility functions are defined as:

\[ v_a = f(t_w, t_g) = \beta_1 + \beta_2 * t_w + \beta_3 * t_g \]  \hspace{1cm} (5)

\[ v_r = 0 \]  \hspace{1cm} (6)

\[ v_a \] Deterministic component of the utility of accepting a gap

\[ v_r \] Deterministic component of the utility of rejecting a gap

\[ t_g \] Gap duration (seconds)

\[ t_w \] Waiting time in the queue and at the stop line (seconds)

\[ \beta_1, \beta_2, \beta_3 \] Parameters

In the logit model, the probability of choosing each alternative depends only on the difference between the utilities. Therefore, by setting the utility of rejecting a gap to zero, the utility of accepting a gap reflects the difference between the utility of accepting and rejecting a gap. The parameters \( \beta \) are estimated by using the maximum likelihood method.

4. RESULTS

4.1 Evaluation of the Results

Table 3 shows the results of the model estimations. The deterministic component of the utility of accepting a gap, therefore is:

\[ v_a = -10.34 + 0.03742 * t_w + 2.509 * t_g \]  \hspace{1cm} (7)

As can be seen from Table 3, all the coefficients are significantly different from zero and the final likelihood is significantly higher than the initial likelihood and also than the likelihood with constants only, showing that the two variables have a significant explanatory power. The results demonstrate as was expected, that the utility from accepting a gap increases as the gap duration increases; according to the logit model therefore, the probability of accepting the gap also increases. As the gap duration increases, the risk in entering the roundabout decreases, and therefore the probability of accepting the gap increases.

Table 3. Results of the Estimation of the Logit Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>(p-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-10.34</td>
<td>-10.7</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Waiting time</td>
<td>0.03742</td>
<td>2.7</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Gap size</td>
<td>2.509</td>
<td>12.1</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Initial likelihood</td>
<td>-515.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Likelihood with constants only</td>
<td>-481.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final likelihood</td>
<td>-148.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td>0.712</td>
</tr>
</tbody>
</table>
The other result was also expected: long waiting times that drivers may experience before facing a gap, causes them to lose patience and, therefore, to be willing to take a greater risk; thus they have a higher probability of accepting a gap. Consequently, a driver’s utility from entering a roundabout increases with higher waiting times. Both coefficients are highly significant and make a significant contribution to the explanatory power of the model.

Figure 1 shows the estimated probabilities of accepting a gap as a function of the waiting time for several different gap values. As can be seen from the Figure, the waiting time mostly affects gaps in the range of 2 to 5 seconds. Gaps lower than two seconds are not likely to be accepted, regardless of the waiting times, as they are considered too risky. Very long waiting time can make a few drivers accept a gap of two seconds. On the other hand, gaps longer than 5 seconds are likely to be accepted immediately, the waiting time has almost no effect. For gaps between 2 and 5 seconds, though, the waiting time can have a significant effect. For example, while the probability of accepting a gap of 3 seconds is 7% after 5 seconds of waiting time, it goes up to 13% after 25 seconds of waiting time and goes up further to 24% after 45 seconds of waiting time.

A comparison of the marginal rate of substitution between the waiting time and the gap duration shows that an addition of 6.7 seconds of waiting time will make a driver willing to accept a gap 0.1 second shorter than the gap that otherwise would have been accepted before this additional waiting time. This trade-off is relevant only within the range of observed gaps in the data; it does not suggest that if waiting time at the approach were much longer, drivers would continue to accept decreasing gaps. This can also be seen from Figure 1.

Figure 2 shows the elasticity of accepting a gap with respect to waiting time. The elasticity of the logit model represents the responsiveness of an individual’s choice probability in regard to a change in the value of some attribute (for a more detailed discussion of how this is derived, see Ben-Akiva and Lerman [1985] pp. 111-113). Figure 2 shows that for large gaps
of 5 and 6 seconds, the elasticity is practically zero, as an increase in waiting time does not change the probability of accepting a gap, which is already high and no longer a function of waiting time. For all gaps, the elasticity starts at zero for zero waiting time, indicating that the first waiting second may not change the probability of accepting a gap. The reason is that drivers often expect that they may have to stop and wait. It is only when the waiting time gets longer that it starts to affect the probability of accepting a gap. With the exception of gaps of 5 seconds and longer, the elasticity increases with waiting time as drivers start to lose their patience and are willing to accept shorter gaps.

![Figure 2. Elasticity of the Probability of Accepting a Gap to Waiting Time](image)

This trend continues for the short gaps of 1-2 second through the whole range of waiting times studied in this analysis (up to 90 seconds). Short gaps are unlikely to be accepted, however, as waiting time increases, the probability of accepting them also continues to increase. For the medium-range gaps of 3-4 seconds, the elasticity has a somewhat parabolic shape and reaches maximum at about 30 seconds of waiting time for the 4-second gap, and at about 65 seconds of waiting time for the 3-second gap; from this point it decreases again. The gaps of 3-4 seconds are those for which waiting time has the most effect. As drivers wait longer, the probability of their accepting a gap increases.

### 4.2 The Critical Gap

The critical gap is defined in this paper as the gap for which the probability of accepting it according to the logit model is equal to 50%. Figure 3 shows the critical gap as a function of waiting time according to this definition and the estimated model of gap-acceptance probability. As can be seen, the critical gap decreases from four seconds for a low waiting time of about 10 seconds to 3 seconds for a long waiting time of 80 seconds.

The critical gap is an input in most entry-capacity models; e.g., HCM, 2000. Therefore, a reduction in critical gaps results in a significant increase in capacity. For example, for a waiting time of 20 seconds and conflicting circulating volumes on the roundabout of 500,
1,000 and 1,500 vehicles per hour, the increase in capacity would be 4.2%, 8.6%, and 13.1%, respectively.

![Logit Model: \( t_{cr} = \frac{(10.34 - 0.037 \times tw)}{2.5} \) @ \( p=0.5 \)](image)

Figure 3: The Critical Gap as a Function of Waiting Time

5. CONCLUSIONS

The paper has presented a new approach to estimate the impact on the critical gap of waiting time prior to entry into a roundabout. Drivers who approach a roundabout face a choice of whether to accept a given gap or to reject it. A disaggregate choice model is utilized to study drivers’ behavior in accepting gaps. Specifically, a binary logit model is used to estimate the effect of waiting time at an approach to a roundabout on the likelihood of accepting different gaps and, therefore, on the critical gap.

The model calibrated showed that the waiting time mostly affects gaps in the range of 2 to 5 seconds. Gaps lower than 2 seconds are not likely to be accepted, regardless of the waiting time, because they are considered too risky. Very long waiting times can cause a few drivers to accept gaps shorter than 2 seconds; on the other hand, gaps longer than 5 seconds are likely to be accepted immediately, so the waiting time has no impact. The marginal rate of substitution between the waiting time and the gap size shows that an increase in the waiting time of 6.7 seconds will make a driver willing to accept a gap 0.1 seconds shorter than the gap he or she would otherwise accept. This finding is applicable to the range of observed gaps in the study as shown in figure 1. An elasticity analysis showed that in roundabout entries, it is the short gaps and long waiting times that are impacted the most from the waiting-time increase, whereas gaps longer than 5 seconds have a zero elasticity, i.e., they are not affected by the waiting times.

Other findings focused on the modeled relationship between the critical gap and the waiting times. The significance and applicability of this model is that it quantitatively shows a reduction in the critical gap with the increase in waiting time. For example, after a waiting
time of 10 seconds, the critical gap would be 4.0 seconds; after a wait of 60 seconds, the critical gap would be 3.25 seconds; this is about a 20% reduction in the critical gap, which would result in a considerable increase in entry capacity. The critical gap is an input in most entry-capacity models; e.g., HCM, 2000. Therefore, the reduction in critical gaps can result in a significant increase in capacity. However, given that the data used were from only seven roundabouts, the results presented here should be considered with caution. Further research is needed to corroborate the findings.

This increase in capacity is significant for busy roundabouts, where waiting times indirectly provide the additional necessary capacity. Further research is suggested on the impact of waiting times on the follow-up time, which may change the entry capacity.

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